Q. Design Turing Machine for language L = {$0^n1^n2^n$ | n≥1}

Solution:

Before designing the required Turing machine M, let us evolve a procedure for processing the input string 112233. After processing, we require the ID to be of the form $bbbbbbq_7$. The processing is done by using five steps:

Step 1 q_1 is the initial state. The R/W head scans the leftmost 1, replaces 1 by b, and moves to the right. M enters q_2.

Step 2 On scanning the leftmost 2, the R/W head replaces 2 by b and moves to the right. M enters q_3.

Step 3 On scanning the leftmost 3, the R/W head replaces 3 by b, and moves to the right. M enters q_4.

Step 4 After scanning the rightmost 3, the R/W heads moves to the left until it finds the leftmost 1. As a result, the leftmost 1, 2 and 3 are replaced by b.

Step 5 Steps 1–4 are repeated until all 1's, 2's and 3's are replaced by blanks.
The change of IDs due to processing of 112233 is given as

$$q_1112233 \vdash bq_212233 \vdash b1q_22233 \vdash b1bq_3233 \vdash b1b2q_333$$

$$\vdash b1b2bq_43 \vdash b1b2q_5b3 \vdash b1bq_52b3 \vdash b1q_5b2b3 \vdash bq_51b2b3$$

$$\vdash q_6b1b2b3 \vdash bq_11b2b3 \vdash bbq_2b2b3 \vdash bbbq_22b3$$

$$\vdash bbbbq_3b3 \vdash bbbbbq_33 \vdash bbbbbbq_4b \vdash bbbbbq_7bb$$

Thus,

$$q_1112233 \overset{*}{\vdash} q_7bbbbbb$$

As q_7 is an accepting state, the input string 112233 is accepted.

Now we can construct the transition table for M.

Present state	Input tape symbol			
	1	2	3	b
$\rightarrow q_1$	bRq_2			bRq_1
q_2	$1Rq_2$	bRq_3		bRq_2
q_3		$2Rq_3$	bRq_4	bRq_3
q_4			$3Lq_5$	bLq_7
q_5	$1Lq_6$	$2Lq_5$		bLq_5
q_6	$1Lq_6$			bRq_1
q_7				

It can be seen from the table that strings other than those of the form $0^n1^n2^n$ are not accepted. It is advisable to compute the computation sequence for strings like 1223, 1123, 1233 and then see that these strings are rejected by M.